Another way to check lin. in/dep (sometimes)

★ $\underline{\text{Thm}}$: If $\underline{A}$ is $\underline{\text{square}}$, then:

1) $\det(\underline{A}) \neq 0 \Rightarrow$ only solution for $\underline{A}\underline{x} = \underline{0}$ is trivial solution $\underline{x} = \underline{0}$

b/c $\underline{x} = \underline{A}^{-1}\underline{0} = \underline{0}$

2) $\det(\underline{A}) = 0 \Rightarrow$ infinitely many solutions for $\underline{A}\underline{x} = \underline{0}$ (incl. $\underline{x} = \underline{0}$)

$\underline{\text{Ex}}$ Are $\underline{u} = \begin{bmatrix} 2 \\ 3 \\ 0 \end{bmatrix}$, $\underline{v} = \begin{bmatrix} 0 \\ 4 \\ 4 \end{bmatrix}$, $\underline{w} = \begin{bmatrix} 4 \\ 2 \\ -2 \end{bmatrix}$ lin dep / indep?

$\longrightarrow$ Check solutions to

$$x_1 \underline{u} + x_2 \underline{v} + x_3 \underline{w} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\underbrace{\begin{bmatrix} \underline{u} & \underline{v} & \underline{w} \end{bmatrix}}_{\underline{A}} \underbrace{\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}}_{\underline{x}} = \underbrace{\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}}_{= \ \underline{0}}$$

$$\underline{A} = \begin{bmatrix} 2 & 0 & 4 \\ 3 & 4 & 2 \\ 0 & 4 & -2 \end{bmatrix} \text{ is square.}$$

$$\det(\underline{A}) = 2(-8-8) - 3(0-16) + 0(\cdots)$$
$$= 16 \neq 0.$$

$\Rightarrow$ only sol. to $\underline{A}\underline{x} = \underline{0}$ (aka $\begin{bmatrix} \underline{u} & \underline{v} & \underline{w} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \underline{0}$)

is trivial sol. $\underline{x} = \underline{0} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$.

So, $\underbrace{\text{columns of } \underline{A}}_{\underline{u},\, \underline{v},\, \underline{w}}$ are linearly $\underline{\text{indep}}$